Due Wed, May 14th, at 11:59pm on Gradescope

Please show your work. Where it makes sense, your solutions should be written in full sentences. Recall that proof-writing problems will be graded on correctness as well as clarity and exposition.

From Enderton:

1. p. 133, Exercises 2, 4, 5 (For #5, we outlined the proof in class. You should fill in the details here.)

   Hint on #4: There are multiple approaches you could take, but here is one. Handle the rational numbers in $(0, 1)$ and $[0, 1]$ differently from the irrational numbers. You might make your one-to-one correspondence be the identity on irrational numbers, and do something interesting on rational numbers. You could also try to take a less direct approach, showing that both $(0, 1) \cap \mathbb{Q}$ and $[0, 1] \cap \mathbb{Q}$ are equinumerous to ω.

2. p. 138, Exercises 6, 7, 8

3. p. 144, Exercises 10, 12, 13 (For #12, please briefly explain.)

Additional problems:

4. (a) Prove that for any natural number n and any set X, either $\text{card}(X) \in n$, or there is a subset $Y \subseteq X$ with $\text{card}(Y) = n$.

   (b) Conclude that if X is an infinite set, then X has subsets of every finite cardinality.

   Note 1: You should not use results beyond p. 138 of the textbook. You may use the results in the section on finite sets.

   Note 2: Be formal and careful with your proof. As a warning: you might think that if X has no finite subsets, then X must have a subset equinumerous with ω. However, this cannot be proven without the Axiom of Choice. (You will not need to use the Axiom of Choice for the proof you are asked to do here.)